GLOBAL EDITION

Digital Control System Analysis and Design

FOURTH EDITION

Charles L. Phillips • H. Troy Nagle • Aranya Chakrabortty

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DIGITAL CONTROL SYSTEM ANALYSIS & DESIGN

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PREFACE

This book is intended to be used primarily as a text for a first course in discrete-time control systems at either the senior undergraduate or first-year graduate level. Furthermore, the text is suitable for self-study by the practicing control engineer.

This book is based on material taught at both Auburn University and North Carolina State University, and in intensive short courses taught in both the United States and Europe. The practicing engineers who attended these short courses have influenced both the content and the direction of this book, resulting in emphasis placed on the practical aspects of designing and implementing digital control systems.

Chapter 1 presents a brief introduction and an outline of the text. Chapters 2–11 cover the analysis and design of discrete-time linear control systems. Some previous knowledge of continuous-time control systems is helpful in understanding this material. The mathematics involved in the analysis and design of discrete-time control systems is the z-transform and vector-matrix difference equations; these topics are presented in Chapter 2. Chapter 3 is devoted to the very important topic of sampling signals and the mathematical model of the sampler and data hold. This model is basic to the remainder of the text. The implications and the limitations of this model are stressed.

The next four chapters, 4–7, are devoted to the application of the mathematics of Chapter 2 to the analysis of discrete-time systems, emphasis on digital control systems. Classical design techniques are covered in Chapter 8, with the frequency-response Bode technique emphasized. Modern design techniques are presented in Chapters 9–11. Chapter 12 summarizes some case studies in discrete-time control system design. Throughout these chapters, practical computer-aided analysis and design using MATLAB are stressed.

In this fourth edition, several changes have been made. We

- Added additional MATLAB examples throughout the chapters.
- Added a new chapter on system identification (Chapter 11).
- Added new problems in many of the chapters.
- Renumbered the end-of-chapter problems to reflect their corresponding textbook sections.
- Added the MATLAB *pidtool* design technique in Chapter 8.
- Added two new case studies in Chapter 12.
- Removed four chapters (formerly Chapters 11–14) and two appendices (formerly Appendices V and VI) on digital filter implementation to reduce the overall page count, thus placing more emphasis on control design.

Each end-of-chapter problem has been written to illustrate basic material in the chapter. Generally, short MATLAB programs are given with many of the textbook examples to illustrate the computer calculations of the results of the example. These programs are easily modified for the homework problems.

To further assist instructors using this book, a set of PowerPoint slides and a manual containing problem solutions has been developed. The authors feel that the problems at the end of the chapters are an indispensable part of the text, and should be fully utilized by all who study this book. Requests for both the problem solutions and PowerPoint slides can be sent directly to the publisher.

At Auburn University, three courses based on the controls portion of this text, Chapters 2–11, have been taught. Chapters 2–8 are covered in their entirety in a one-quarter four-credit-hour graduate course. Thus the material is also suitable for a three-semester-hour course and has been presented as such at North Carolina State University. These chapters have also been covered in twenty lecture hours of an undergraduate course, but with much of the material omitted. The topics not covered in this abbreviated presentation are state variables, the modified z-transform, nonsynchronous sampling, and closed-loop frequency response. A third course, which is a one-quarter three-credit course, requires one of the above courses as a prerequisite, and introduces the state variables of Chapter 2. Then the state-variable models of Chapter 4, and the modern design of Chapters 9–11, are covered in detail. In a recent offering at North Carolina State University, Chapters 2–11 were covered in a one-semester, three-credit-hour course using this new edition and the companion set of PowerPoint slides that are also available.

Finally, we gratefully acknowledge the many colleagues, graduate and undergraduate students, and staff members of the Electrical Engineering Department at Auburn University who contributed to the development of the first three editions of this book. In particular, we wish to thank Professor J. David Irwin, Electrical Engineering Department Head at Auburn University, for his aid and encouragement during those years. We would also like to acknowledge our colleagues and students in the Electrical and Computer Engineering Department at North Carolina State University for their contributions to and support for this fourth edition.

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1

Introduction

1.1 OVERVIEW

This book is concerned with the analysis and design of closed-loop physical systems that contain digital computers. The computers are placed within the system to modify the dynamics of the closed-loop system such that a *more satisfactory* system response is obtained.

A *closed-loop system* is one in which certain system forcing functions (inputs) are determined, at least in part, by the response (outputs) of the system (i.e., the input is a function of the output). A simple closed-loop system is illustrated in Fig. 1-1. The physical system (process) to be controlled is called the *plant.* Usually a system, called the *control actuator,* is required to drive the plant; in Fig. 1-1 the actuator has been included in the plant. The *sensor* (or sensors) measures the response of the plant, which is then compared to the desired response. This difference signal initiates actions that result in the actual response approaching the desired response, which drives the difference signal toward zero. Generally, an unacceptable closed-loop response occurs if the plant input is simply the difference between the desired response and actual response. Instead, this difference signal must be processed (filtered) by another physical system, which is called a *compensator,* a *controller,* or simply a *filter.* One problem of the control system designer is the design of the compensator.

An example of a closed-loop system is the case of a pilot landing an aircraft. For this example, in Fig. 1-1 the plant is the aircraft and the plant inputs are the pilot's manipulations of the various control surfaces and of the aircraft velocity. The pilot is the sensor, with his or her visual perceptions of position, velocity, instrument indications, and so on, and with his or her sense of balance, motion, and so on. The desired response is the pilot's concept of the desired flight path. The compensation is the pilot's manner of correcting perceived errors in flight path. Hence, for this example, the compensation, the sensor, and the generation of the desired response are all functions performed by the pilot. It is obvious from this example that the compensation must be a function of plant (aircraft) dynamics. A pilot trained only in a fighter aircraft is not qualified to land a large passenger aircraft, even if he or she can manipulate the controls.

We will consider systems of the type shown in Fig. 1-1, in which the sensor is an appropriate measuring instrument and the compensation function is performed by a digital computer.

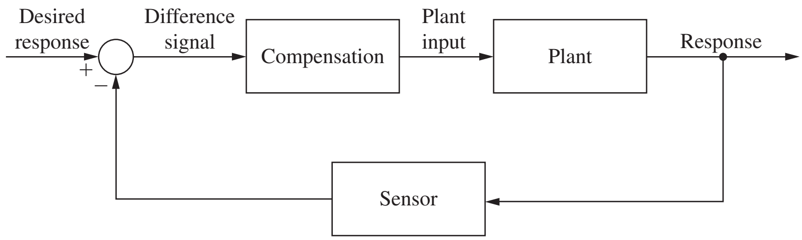


FIGURE 1-1 Closed-loop system.

The plant has dynamics; we will program the computer such that it has *dynamics* of the same nature as those of the plant. Furthermore, although generally we cannot choose the dynamics of the plant, we can choose those of the computer such that, in some sense, the dynamics of the closed-loop system are *satisfactory*. For example, if we are designing an automatic aircraft landing system, the landing must be safe, the ride must be acceptable to the pilot and to any passengers, and the aircraft cannot be unduly stressed.

Both classical and modern control techniques of analysis and design are developed in this book. Almost all control-system techniques developed are applicable to *linear time-invariant discrete-time* system models. A linear system is one for which the principle of superposition applies [1]. Suppose that the input of a system $x_1(t)$ produces a response (output) $y_1(t)$, and the input $x_2(t)$ produces the response $y_2(t)$. Then, if the system is linear, the principle of superposition applies and the input $[a_1x_1(t) + a_2x_2(t)]$ will produce the output $[a_1y_1(t) + a_2y_2(t)]$, where a_1 and a_2 are any constants. All physical systems are inherently nonlinear; however, in many systems, if the system signals vary over a narrow range, the system responds in a linear manner. Even though the analysis and design techniques presented are applicable to linear systems only, certain nonlinear effects will be discussed.

When the parameters of a system are constant with respect to time, the system is called a *time-invariant* system. An example of a time-varying system is the booster stage of a space vehicle, in which fuel is consumed at a known rate; for this case, the mass of the vehicle decreases with time.

A *discrete-time* system has signals that can change values only at discrete instants of time. We will refer to systems in which all signals can change continuously with time as *continuous-time,* or *analog,* systems.

The compensator, or controller, in this book is a digital filter. The filter implements a transfer function. The design of transfer functions for digital controllers is the subject of Chapters 2 through 9 and 11. Once the transfer function is known, algorithms for its realization must be programmed on a digital computer. In Chapter 10 we introduce system identification methods to model the plant's dynamic behavior. In Chapter 12 we present several case studies in digital controls systems design.

Presented next in this chapter is an example digital control system. Then the equations describing three typical plants that appear in closed-loop systems are developed.

1.2 DIGITAL CONTROL SYSTEM

The basic structure of a digital control system will be introduced through the example of an automatic aircraft landing system. The system to be described is similar to the landing system that is currently operational on U.S. Navy aircraft carriers [2]. Only the simpler aspects of the system will be described.

The automatic aircraft landing system is depicted in Fig. 1-2. The system consists of three basic parts: the aircraft, the radar unit, and the controlling unit. During the operation of this control system, the radar unit measures the approximate vertical and lateral positions of the aircraft, which are then transmitted to the controlling unit. From these measurements, the controlling unit calculates appropriate pitch and bank commands. These commands are then transmitted to the aircraft autopilots, which in turn cause the aircraft to respond accordingly.

In Fig. 1-2 the controlling unit is a digital computer. The lateral control system, which controls the lateral position of the aircraft, and the vertical control system, which controls the altitude of the aircraft, are independent (decoupled). Thus the bank command input affects only the lateral position of the aircraft, and the pitch command input affects only the altitude of the aircraft. To simplify the treatment further, only the lateral control system will be discussed.

A block diagram of the lateral control system is given in Fig. 1-3. The aircraft lateral position, $y(t)$, is the lateral distance of the aircraft from the extended centerline of the runway. The control system attempts to force $y(t)$ to zero. The radar unit measures $y(t)$ every 0.05 s. Thus $y(kT)$ is the sampled value of $y(t)$, with $T = 0.05$ s and $k = 0, 1, 2, 3, \ldots$. The digital controller processes these sampled values and generates the discrete bank commands $\phi(kT)$. The data hold, which is on board the aircraft, clamps the bank command $\phi(t)$ constant at the last value received until the next value is received. Then the bank command is held constant at the new value until the following value is received. Thus the bank command is updated every $T = 0.05$ s, which is called the *sample period.* The aircraft responds to the bank command, which changes the lateral position $y(t)$.

Two additional inputs are shown in Fig. 1-3. These are unwanted inputs, called *disturbances,* and we would prefer that they not exist. The first, $w(t)$, is the wind input, which certainly affects the position of the aircraft. The second disturbance input, labeled radar noise, is present since the radar cannot measure the exact position of the aircraft. This noise is the difference

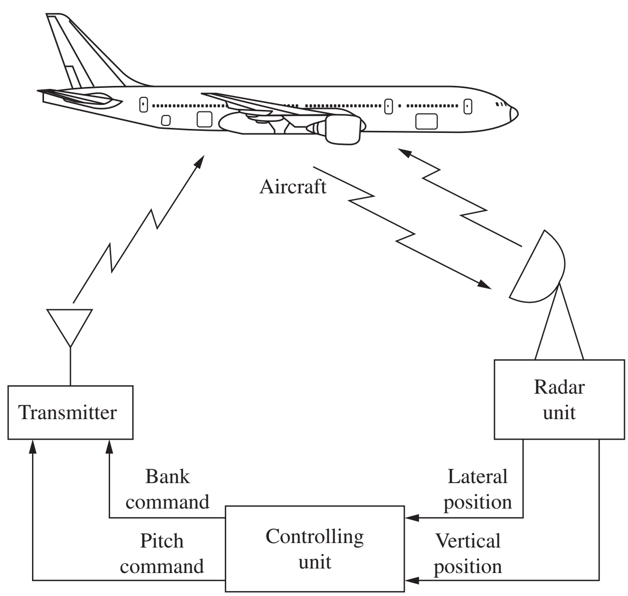
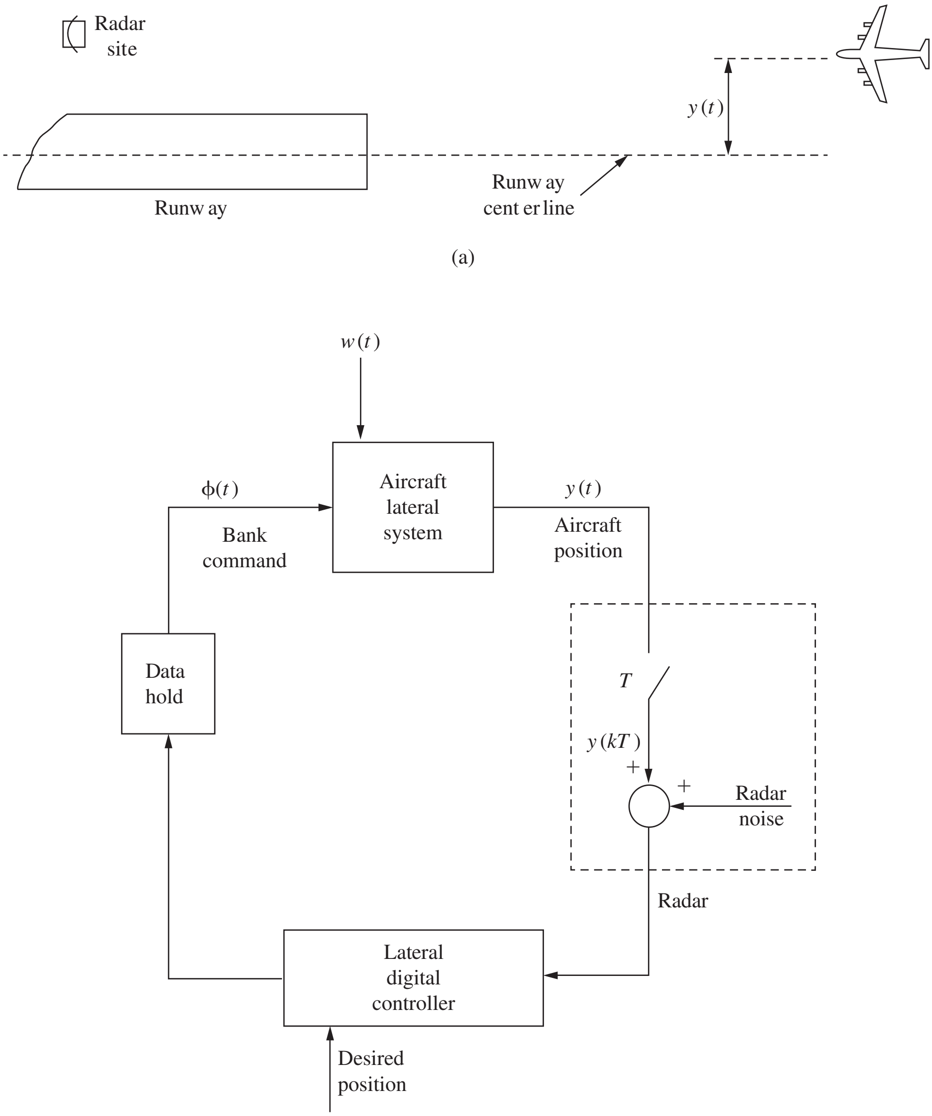


FIGURE 1-2 Automatic aircraft landing system.



(b)

FIGURE 1-3 Aircraft lateral control system.

between the exact aircraft position and the measured position. Since no sensor is perfect, sensor noise is always present in a control system.

The design problem for this system is to maintain $y(t)$ at a small level in the presence of the wind and radar-noise disturbances. In addition, the plane must respond in a manner that both is acceptable to the pilot and does not unduly stress the structure of the aircraft.

To effect the design, it is necessary to know the mathematical relationships between the wind input $w(t)$, the bank command input $\phi(t)$, and the lateral position $y(t)$. These mathematical relationships are referred to as the mathematical model, or simply the model, of the aircraft. For example, for the McDonnell-Douglas Corporation F4 aircraft, the model of lateral system is a ninth-order ordinary nonlinear differential equation [3]. For the case in which the bank command $\phi(t)$ remains small in amplitude, the nonlinearities are not excited and the system model described by this ninth-order ordinary nonlinear differential equation may be used for design purposes.

The task of the control system designer is to specify the processing to be accomplished in the digital controller. This processing will be a function of the ninth-order aircraft model, the expected wind input, the radar noise, the sample period T, and the desired response characteristics. Various methods of digital controller design are developed in Chapters 8, 9, and 11.

The development of the ninth-order model of the aircraft is beyond the scope of this book. In addition, this model is too complex to be used in an example in this book. Hence, to illustrate the development of models of physical systems, the mathematical models of four simple, but common, control-system plants will be developed later in this chapter. Two of the systems relate to the control of position, the third relates to temperature control, and the fourth one describes control of electrical power in single-machine infinite bus models of power systems. In addition, Chapter 10 presents procedures for determining the model of a physical system from input–output measurements of the system.

1.3 THE CONTROL PROBLEM

We may state the control problem as follows. A physical system or process is to be accurately controlled through closed-loop, or feedback, operation. An output variable (signal), called the response, is adjusted as required by an error signal. The error signal is a measure of the difference between the system response, as determined by a sensor, and the desired response.

Generally, a controller, or filter, is required to process the error signal in order that certain control criteria, or specifications, will be satisfied. The criteria may involve, but not be limited to:

1. Disturbance rejection
2. Steady-state errors
3. Transient response
4. Sensitivity to parameter changes in the plant

Solving the control problem will generally involve:

1. Choosing sensors to measure the required feedback signals
2. Choosing actuators to drive the plant
3. Developing the plant, sensor, and actuator models (equations)
4. Designing the controller based on the developed models and the control criteria
5. Evaluating the design analytically, by simulation, and finally, by testing the physical system
6. Iterating this procedure until a satisfactory physical-system response results

Because of inaccuracies in the mathematical models, the initial tests on the physical system may not be satisfactory. The controls engineer must then iterate this design procedure, using *all* tools available, to improve the system. Intuition, developed while experimenting with the physical system, usually plays an important part in the design process.

Fig. 1-4 illustrates the relationship of mathematical analysis and design to physical-system design procedures [4]. In this book, all phases shown in the Fig. are discussed, but the emphasis is necessarily on the conceptual part of the procedures—the application of mathematical concepts to mathematical models. In practical design situations, however, the major difficulties are in formulating the problem mathematically and in translating the mathematical solution back to the physical world. Many iterations of the procedures shown in Fig. 1-4 are usually required in practical situations.

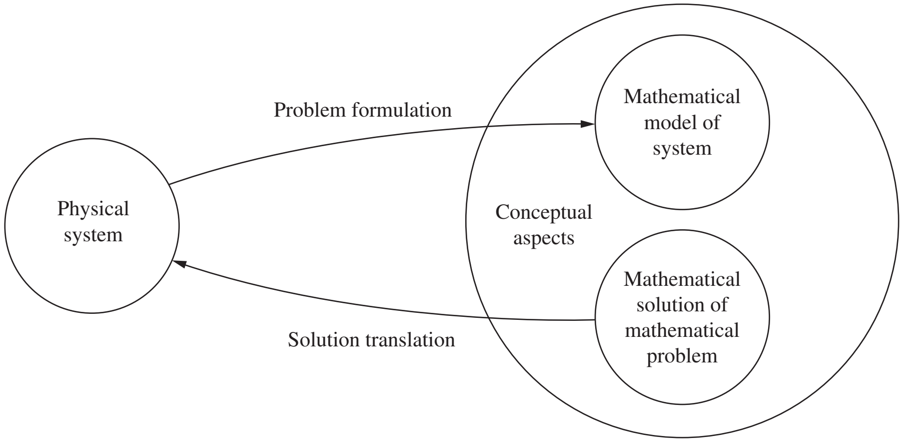


FIGURE 1-4 Mathematical solutions for physical systems.

Depending on the system and the experience of the designer, some of the steps listed earlier may be omitted. In particular, many control systems are implemented by choosing standard forms of controllers and experimentally determining the parameters of the controller; a specified step-by-step procedure is applied directly to the physical system, and no mathematical models are developed. This type of procedure works very well for certain control systems. For other systems, it does not. For example, a control system for a space vehicle cannot be designed in this manner; this system must perform satisfactorily the first time it is activated.

In this book mathematical procedures are developed for the analysis and design of control systems. The techniques developed may or may not be of value in the design of a particular control system. However, standard controllers are utilized in the developments in this book. Thus the analytical procedures develop the concepts of control system design and indicate applications of each of the standard controllers.

1.4 SATELLITE MODEL

As the first example of the development of the mathematical model of a physical system, we will consider the attitude control system of a satellite. Assume that the satellite is spherical and has the thruster configuration shown in Fig. 1-5. Suppose that $\theta(t)$ is the yaw angle of the satellite. In addition to the thrusters shown, thrusters will also control the pitch angle and the roll angle, giving complete three-axis control of the satellite. We will consider only the yaw-axis control systems, whose purpose is to control the angle $\theta(t)$.

For the satellite, the thrusters, when active, apply a torque $\tau(t)$. The torque of the two active thrusters shown in Fig. 1-5 tends to reduce $\theta(t)$. The other two thrusters shown tend to increase $\theta(t)$.

Since there is essentially no friction in the environment of a satellite, and assuming the satellite to be rigid, we can write

$$J\frac{d^2\theta(t)}{dt^2} = \tau(t) \tag{1-1}$$

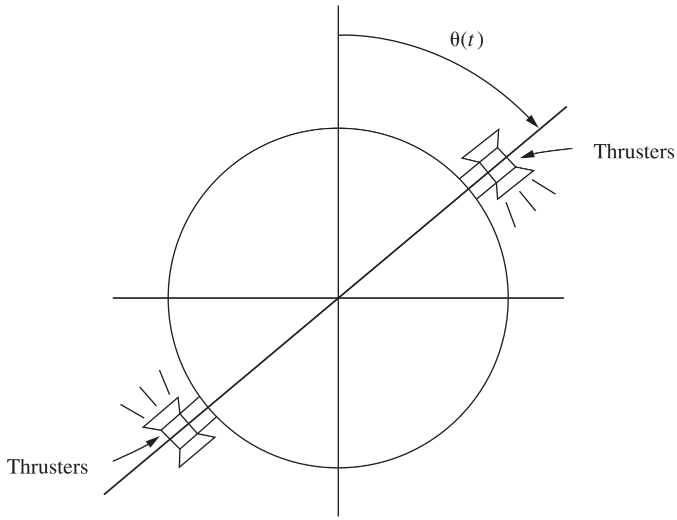


FIGURE 1-5 Satellite.

where J is the satellite's moment of inertia about the yaw axis. We now derive the transfer function by taking the Laplace transform of (1-1):

$$Js^2\Theta(s) = T(s) = \mathcal{L}[\tau(t)] \tag{1-2}$$

Initial conditions are ignored when deriving transfer functions. Equation (1-2) can be expressed as

$$\frac{\Theta(s)}{T(s)} = G_p(s) = \frac{1}{Js^2} \tag{1-3}$$

The ratio of the Laplace transforms of the output variable $[\theta(t)]$ to input variable $[\tau(t)]$ is called the plant transfer function, and is denoted here as $G_p(s)$. A brief review of the Laplace transform is given in Appendix V.

The model of the satellite may be specified by either the second-order *differential equation* of (1-1) or the second-order *transfer function* of (1-3). A third model is the state-variable model, which we will now develop. Suppose that we define the variables $x_1(t)$ and $x_2(t)$ as

$$x_1(t) = \theta(t) \tag{1-4}$$

$$x_2(t) = \dot{x}_1(t) = \dot{\theta}(t) \tag{1-5}$$

where $\dot{x}_1(t)$ denotes the derivative of $x_1(t)$ with respect to time. Then, from (1-1) and (1-5),

$$\dot{x}_2(t) = \ddot{\theta}(t) = \frac{1}{J}\tau(t) \tag{1-6}$$

where $\ddot{\theta}(t)$ is the second derivative of $\theta(t)$ with respect to time.

We can now write (1-5) and (1-6) in vector-matrix form (see Appendix IV):

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} \tau(t) \qquad \textbf{(1-7)}$$

In this equation, $x_1(t)$ and $x_2(t)$ are called the state variables. Hence we may specify the model of the satellite in the form of (1-1), or (1-3), or (1-7). State-variable models of analog systems are considered in greater detail in Chapter 4.

1.5 SERVOMOTOR SYSTEM MODEL

In this section the model of a servo system (a positioning system) is derived. An example of this type of system is an antenna tracking system. In this system, an electric motor is utilized to rotate a radar antenna that tracks an aircraft automatically. The error signal, which is proportional to the difference between the pointing direction of the antenna and the line of sight to the aircraft, is amplified and drives the motor in the appropriate direction so as to reduce this error.

A dc motor system is shown in Fig. 1-6. The motor is armature controlled with a constant field. The armature resistance and inductance are R_a and L_a, respectively. We assume that the inductance L_a can be ignored, which is the case for many servomotors. The motor back emf $e_m(t)$ is given by [5]

$$e_m(t) = K_b\omega(t) = K_b\frac{d\theta(t)}{dt} \qquad \textbf{(1-8)}$$

where $\theta(t)$ is the shaft position, $\omega(t)$ is the shaft angular velocity, and K_b is a motor-dependent constant. The total moment of inertia connected to the motor shaft is J, and B is the total viscous friction. Letting $\tau(t)$ be the torque developed by the motor, we write

$$\tau(t) = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} \qquad \textbf{(1-9)}$$

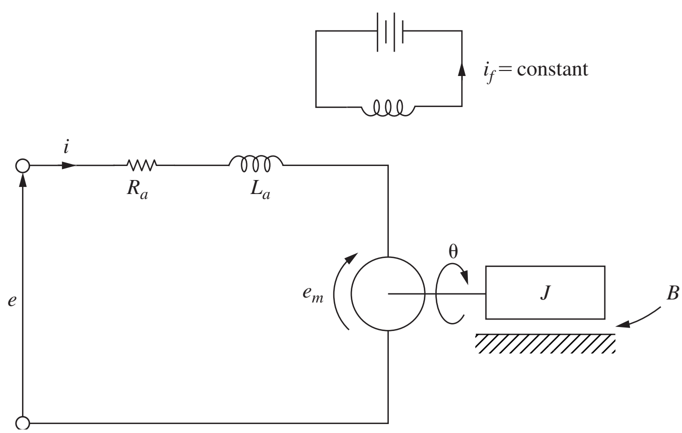


FIGURE 1-6 Servomotor system.

The developed torque for this motor is given by

$$\tau(t) = K_T i(t) \tag{1-10}$$

where $i(t)$ is the armature current and K_T is a parameter of the motor. The final equation required is the voltage equation for the armature circuit:

$$e(t) = i(t)R_a + e_m(t) \tag{1-11}$$

These four equations may be solved for the output $\theta(t)$ as a function of the input $e(t)$. First, from (1-11) and (1-8),

$$i(t) = \frac{e(t) - e_m(t)}{R_a} = \frac{e(t)}{R_a} - \frac{K_b}{R_a}\frac{d\theta(t)}{dt} \tag{1-12}$$

Then, from (1-9), (1-10), and (1-12),

$$\tau(t) = K_T i(t) = \frac{K_T}{R_a}e(t) - \frac{K_T K_b}{R_a}\frac{d\theta(t)}{dt} = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} \tag{1-13}$$

This equation may be written as

$$J\frac{d^2\theta(t)}{dt^2} + \frac{BR_a + K_T K_b}{R_a}\frac{d\theta(t)}{dt} = \frac{K_T}{R_a}e(t) \tag{1-14}$$

which is the desired model. This model is second order; if the armature inductance cannot be neglected, the model is third order [6].

Next we take the Laplace transform of (1-14) and solve for the transfer function:

$$\frac{\Theta(s)}{E(s)} = G_p(s) = \frac{K_T/R_a}{Js^2 + \dfrac{BR_a + K_T K_b}{R_a}s} = \frac{K_T/JR_a}{s\left(s + \dfrac{BR_a + K_T K_b}{JR_a}\right)} \tag{1-15}$$

Many of the examples of this book are based on this transfer function.

The state-variable model of this system is derived as in the preceding section. Let

$$\begin{aligned} x_1(t) &= \theta(t) \\ x_2(t) &= \dot{\theta}(t) = \dot{x}_1(t) \end{aligned} \tag{1-16}$$

Then, from (1-14),

$$\dot{x}_2(t) = \ddot{\theta}(t) = -\frac{BR_a + K_T K_b}{JR_a}x_2(t) + \frac{K_T}{JR_a}e(t) \tag{1-17}$$

Hence the state equations may be written as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{BR_a + K_T K_b}{JR_a} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{K_T}{JR_a} \end{bmatrix} e(t) \tag{1-18}$$

Antenna Pointing System

We define a *servomechanism,* or more simply, a *servo,* as a system in which mechanical position is controlled. Two servo systems, which in this case form an antenna pointing system, are illustrated in Fig. 1-7. The top view of the pedestal illustrates the *yaw-axis* control system. The yaw angle, $\theta(t)$, is controlled by the electric motor and gear system (the control actuator) shown in the side view of the pedestal.

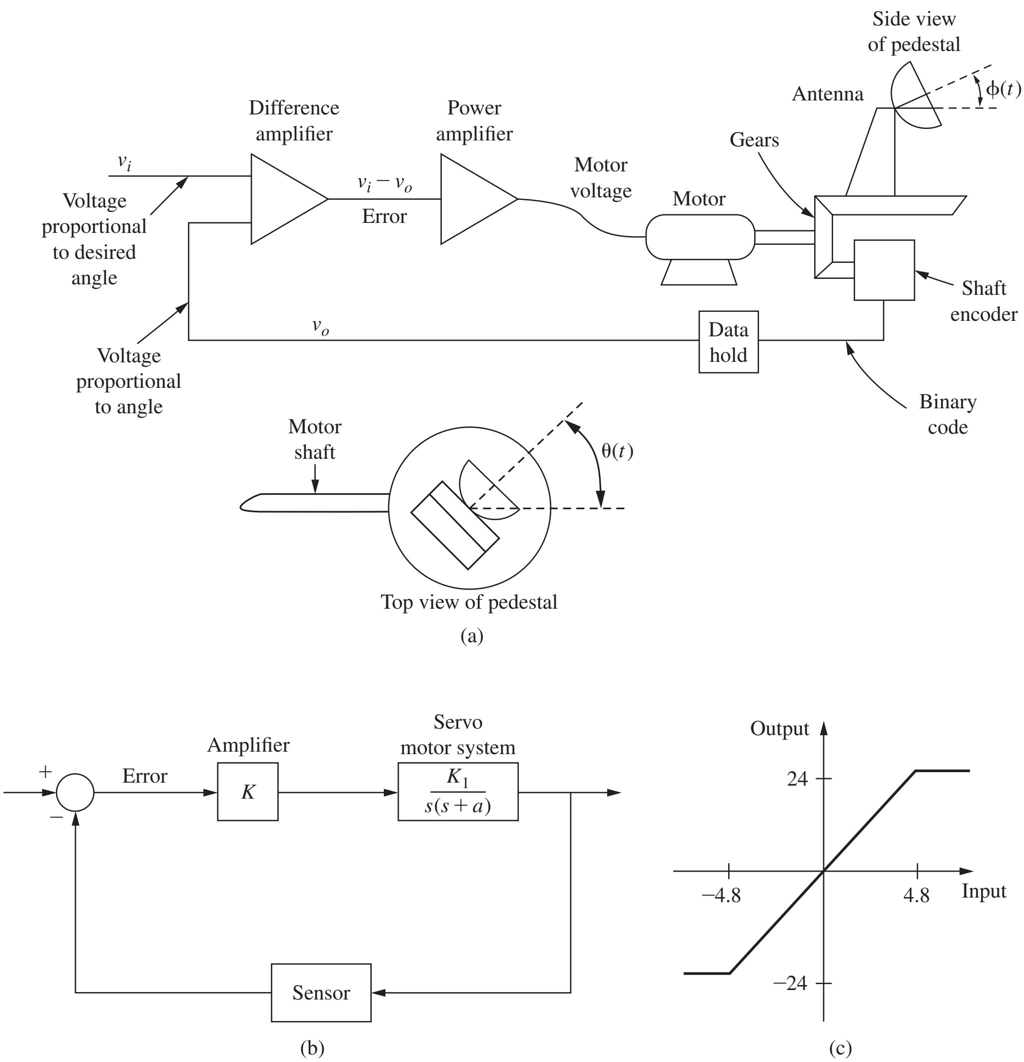


FIGURE 1-7 Servo control system.

The pitch angle, $\phi(t)$, is shown in the side view. This angle is controlled by a motor and gear system within the pedestal; this actuator is not shown.

We consider only the yaw-axis control system. The electric motor rotates the antenna and the sensor, which is a digital shaft encoder [7]. The output of the encoder is a binary number that is proportional to the angle of the shaft. For this example, a digital-to-analog converter is used to convert the binary number to a voltage $v_o(t)$ that is proportional to the angle of rotation of the shaft. Later we consider examples in which the binary number is transmitted directly to a digital controller.

In Fig. 1-7(a) the voltage $v_o(t)$ is directly proportional to the yaw angle of the antenna, and the voltage $v_i(t)$ is directly proportional (same proportionality constant) to the desired yaw angle. If the actual yaw angle and the desired yaw angle are different, the error voltage $e(t)$ is nonzero. This voltage is amplified and applied to the motor to cause rotation of the motor shaft in the direction that reduces the error voltage.

The system *block diagram* is given in Fig. 1-7(b). Since the error signal is normally a low-power signal, a power amplifier is required to drive the motor. However, this amplifier introduces a nonlinearity, since an amplifier has a maximum output voltage and can be saturated at this value. Suppose that the amplifier has a gain of 5 and a maximum output of 24 V. Then the amplifier input–output characteristic is as shown in Fig. 1-7(c). The amplifier saturates at an input of 4.8 V; hence, for an error signal larger than 4.8 V in magnitude, the system is nonlinear.

In many control systems, we go to great lengths to ensure that the system operation is confined to linear regions. In other systems, we purposely design for nonlinear operation. For example, in this servo system, we must apply maximum voltage to the motor to achieve maximum speed of response. Thus for large error signals we would have the amplifier saturated in order to achieve a fast response.

The analysis and design of nonlinear systems is beyond the scope of this book; we will always assume that the system under consideration is operating in a linear mode.

Robotic Control System

A line drawing of an industrial robot is shown in Fig. 1-8. The basic element of the control system for each joint of many robots is a servomotor. We take the usual approach of considering each joint of the robot as a simple servomechanism, and ignore the movements of the other joints in the arm. Although this approach is simple in terms of analysis and design, the result is often less than desirable control of the joint [8].

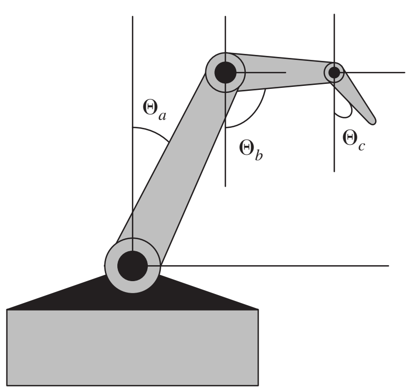


FIGURE 1-8 Schematic diagram of a robotic arm with three angles of motion.

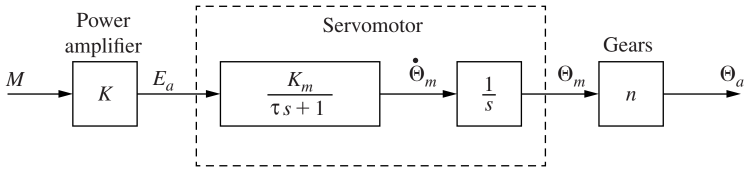


FIGURE 1-9 Model of robot arm joint.

The model of a single robot arm joint is given in Fig. 1-9, where the second-order model of the servomotor is assumed. In addition, it is assumed that the arm is attached to the motor through gears, with a gear ratio of n. If the armature inductance of the motor cannot be ignored, the model is third order [8]. In this model, $E_a(s)$ is the armature voltage, and is used to control the position of the arm. The input signal $M(s)$ is assumed to be from a digital computer, and the power amplifier K is required since a computer output signal cannot drive the motor. The angle of the motor shaft is $\Theta_m(s)$, and the angle of the arm is $\Theta_a(s)$. Same holds for the two other arm angles $\Theta_b(s)$ and $\Theta_c(s)$. As described above, the inertia and friction of both the gears and the arm are included in the servomotor model, and hence the model shown is the complete model of the robot joint. This model will be used in several problems that appear at the ends of the chapters.

1.6 TEMPERATURE CONTROL SYSTEM

As a third example of modeling, a thermal system will be considered. It is desired to control the temperature of a liquid in a tank. Liquid is flowing out at some rate, being replaced by liquid at temperature $\tau_i(t)$ as shown in Fig. 1-10. A mixer agitates the liquid such that the liquid temperature can be assumed uniform at a value $\tau(t)$ throughout the tank. The liquid is heated by an electric heater.

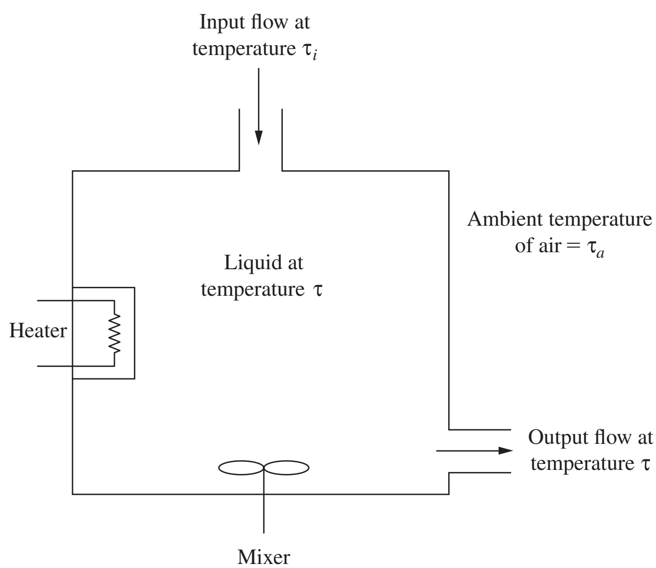


FIGURE 1-10 Thermal system.

We first make the following definitions:

$$
\begin{aligned}
q_e(t) &= \text{heat flow supplied by the electric heater} \\
q_i(t) &= \text{heat flow via liquid entering the tank} \\
q_l(t) &= \text{heat flow into the liquid} \\
q_o(t) &= \text{heat flow via liquid leaving the tank} \\
q_s(t) &= \text{heat flow through the tank surface}
\end{aligned}
$$

By the conservation of energy, heat added to the tank must equal that stored in the tank plus that lost from the tank. Thus

$$q_e(t) + q_i(t) = q_l(t) + q_o(t) + q_s(t) \qquad \textbf{(1-19)}$$

Now [9]

$$q_l(t) = C\frac{d\tau(t)}{dt} \qquad \textbf{(1-20)}$$

where C is the thermal capacity of the liquid in the tank. Letting $v(t)$ equal the flow into and out of the tank (assumed equal) and H equal the specific heat of the liquid, we can write

$$q_i(t) = v(t)H\tau_i(t) \qquad \textbf{(1-21)}$$

and

$$q_o(t) = v(t)H\tau(t) \qquad \textbf{(1-22)}$$

Let $\tau_a(t)$ be the ambient temperature outside the tank and R be the thermal resistance to heat flow through the tank surface. Then

$$q_s(t) = \frac{\tau(t) - \tau_a(t)}{R} \qquad \textbf{(1-23)}$$

Substituting (1-20) through (1-23) into (1-19) yields

$$q_e(t) + v(t)H\tau_i(t) = C\frac{d\tau(t)}{dt} + v(t)H\tau(t) + \frac{\tau(t) - \tau_a(t)}{R}$$

We now make the assumption that the flow $v(t)$ is constant with the value V; otherwise, the last differential equation is time-varying. Then

$$q_e(t) + VH\tau_i(t) = C\frac{d\tau(t)}{dt} + VH\tau(t) + \frac{\tau(t) - \tau_a(t)}{R} \qquad \textbf{(1-24)}$$

This model is a first-order linear differential equation with constant coefficients. In terms of a control system, $q_e(t)$ is the control input signal, $\tau_i(t)$ and $\tau_a(t)$ are disturbance input signals, and $\tau(t)$ is the output signal.

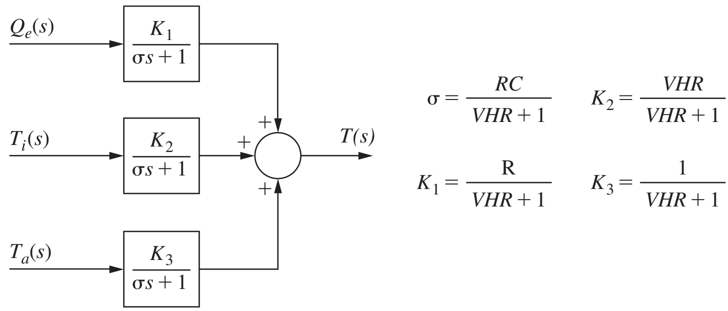


FIGURE 1-11 Block diagram of a thermal system.

Taking the Laplace of (1-24) and solving for $T(s) = \mathscr{L}[\tau(t)]$ yields

$$T(s) = \frac{Q_e(s)}{Cs + VH + (1/R)} + \frac{VHT_i(s)}{Cs + VH + (1/R)} + \frac{(1/R)\,T_a(s)}{Cs + VH + (1/R)} \tag{1-25}$$

Different configurations may be used to express (1-25) as a block diagram; one is given in Fig. 1-11.

If we ignore the disturbance inputs, the transfer-function model of the system is simple and first order. However, at some step in the control system design the disturbances must be considered. Quite often a major specification in a control system design is the minimization of system response to disturbance inputs.

The model developed in this section also applies directly to the control of the air temperature in an oven or a test chamber. For many of these systems, no air is introduced from the outside; hence the disturbance input $q_i(t)$ is zero. Of course, the parameters for the liquid in (1-25) are replaced with those for air.

1.7 SINGLE-MACHINE INFINITE BUS POWER SYSTEM

A single-machine infinite bus (SMIB) power system, as shown in Fig. 1-12, is often used as the starting model for understanding dynamics and stability of large power grids. The system consists of a synchronous generator *G,* which in many cases may represent the equivalent model of a larger area containing multiple synchronous machines inside it, supplying electrical power across a lossless transmission line to a load connected to a fixed or stationary point, commonly referred to as the infinite bus. The relevance of the term "infinite" is that this bus can also be viewed as a generator with theoretically infinite inertia, implying that the voltage and phase

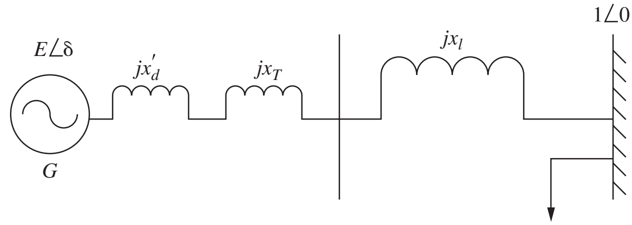


FIGURE 1-12 Single-machine infinite bus power system.

angle at this bus always remain static or constant, and thereby serve as a reference for quantitative analysis of the phase angle oscillations of the other generator(s) in the system. Therefore, for convenience, it is always assumed that the voltage at the infinite bus is 1 per unit, while the phase angle is 0 degrees. To derive the model of the SMIB power system, we next introduce the following set of symbols:

δ phase angle of the synchronous generator (radians)

ω angular speed (radian/sec)

ω_s synchronous speed, equal to 120π rad/s for a 60 Hz system

E internal constant voltage of the generator (per unit)

x'_d direct-axis salient reactance (per unit)

x_T transformer reactance (per unit)

x_l transmission line reactance (per unit)

d damping constant

M generator electro-mechanical inertia

P_m mechanical power input from turbine to generator (per unit)

P_e electrical power output from generator to infinite bus (per unit)

For details of the physical meanings of the above notations please see [10]. The dynamic model of the SMIB system can be written by applying Newton's second law of motion as

$$\begin{aligned} \dot{\delta} &= \omega - \omega_s \\ M\dot{\omega} &= P_m - P_e - d\omega \end{aligned} \tag{1-26}$$

which implies that the imbalance between input and output power flow causes the rotor of the synchronous generator to accelerate. From electric circuit law, the total complex power flowing from G to the infinite bus can be written as

$$P = (E\angle\delta)\widetilde{I}^{*} \tag{1-27}$$

where, $E\angle\delta = E(\cos\delta + j\sin\delta)$, $\widetilde{I}$ is the current phasor which is flowing out of the machine, and the superscript * means complex conjugate. From Kirchoff's law this current can be written as

$$\widetilde{I} = \frac{E\angle\delta - 1\angle 0}{j(x'_d + x_T + x_l)} \tag{1-28}$$

where $1\angle 0$ is the voltage phasor at the infinite bus, while the expression in the denominator denotes the total reactance between the generator and the infinite bus. For simplicity of notation let us denote $x = x'_d + x_T + x_l$. Then the expression for the current phasor can be simplified as

$$\widetilde{I} = \frac{E\cos\delta - 1 + jE\sin\delta}{jx} \tag{1-29}$$

from which P, after a few simple calculations, can be shown as

$$P = \frac{E}{x}(\sin\delta + j(E - \cos\delta)) \tag{1-30}$$

Since P_e in (1-26) represents only the real part of P, therefore another way to write (1-26) is

$$\begin{aligned} \dot{\delta} &= \omega - \omega_s \\ M\dot{\omega} &= P_m - \frac{E}{x}\sin\delta - d\omega \end{aligned} \quad \textbf{(1-31)}$$

Equation (1-31) gives the continuous-time state-variable model for the SMIB system. The model, however, is nonlinear because of the sin δ term in the RHS of the second equation. Hence, we linearize this model across an equilibrium of ($\delta = \delta_0, \omega = \omega_s$) to obtain a linear time-invariant model of the form

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{E\cos\delta_0}{Mx} & -\dfrac{d}{M} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{M} \end{bmatrix} \Delta P_m \quad \textbf{(1-32)}$$

where Δ stands for the small-signal perturbation of the corresponding states and inputs from their respective equilibrium values. The output can be considered as the change in electric power P_e as

$$\Delta P_e = \frac{E\cos\delta_0}{x}\Delta\delta = k\Delta\delta \quad \textbf{(1-33)}$$

The transfer function for the system (1-32) and (1-33) with input ΔP_m and output ΔP_e can be derived as

$$G_p(s) = \frac{k}{Ms^2 + ds + k} \quad \textbf{(1-34)}$$

$G_p(s)$ gives the open-loop transfer function between the small-signal mechanical power input and the electrical power output of the synchronous machine. It can be seen that the steady-state gain ($s = 0$) of $G_p(s)$ is 1, which means that in steady state the mechanical power input to the machine must exactly balance its electrical power output. The transient response of the output for a unit change in the input, however, may not be satisfactory to the user as it is. Therefore, one may design an output-feedback controller $C(s)$ to control the transient response of the electrical power, as shown in the block diagram in Fig. 1-13. $C(s)$ must be designed so that the steady-state gain of the closed-loop transfer function is 1. Depending on the values of M, d, and k, the open-loop response may not be satisfactory in terms of damping, percent peak overshoot, settling time, etc. The controller $C(s)$ can be designed to achieve these transient performance specifications.

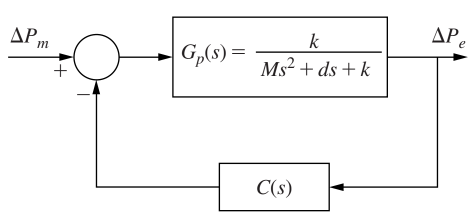


FIGURE 1-13 Block diagram of a closed-loop SMIB power system model.

1.8 SUMMARY

In this chapter we have introduced the concepts of a closed-loop control system. Next, models of four physical systems were discussed. First, a model of a satellite was derived. Next, the model of a servomotor was developed; then two examples, an antenna pointing system and a robot arm, were discussed. Next, a model was developed for control of the temperature of a tank of liquid. Finally, a model for a single-machine infinite bus (SMIB) power system was presented. These systems are continuous time, and generally, the Laplace transform is used in the analysis and design of these systems. In the next chapter we extend the concepts of this chapter to a system controlled by a digital computer and introduce some of the mathematics required to analyze and design this type of system.

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Problems

1.1-1. (a) Show that the transfer function of two systems in parallel, as shown in Fig. P1.1-l(a), is equal to the sum of the transfer functions.

(b) Show that the transfer function of two systems in series (cascade), as shown in Fig. Pl.1-l(b), is equal to the product of the transfer functions.

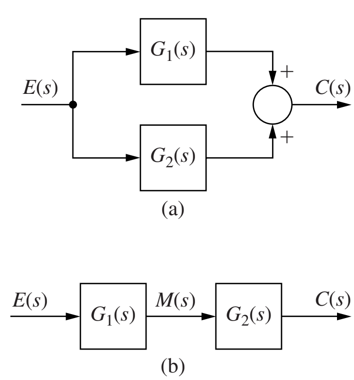


FIGURE P1.1-1 Systems for Problem 1.1-1.

1.1-2. By writing algebraic equations and eliminating variables, calculate the transfer function $C(s)/R(s)$ for the system of:

(a) Figure P1.1-2(a).
(b) Figure P1.1-2(b).
(c) Figure P1.1-2(c).

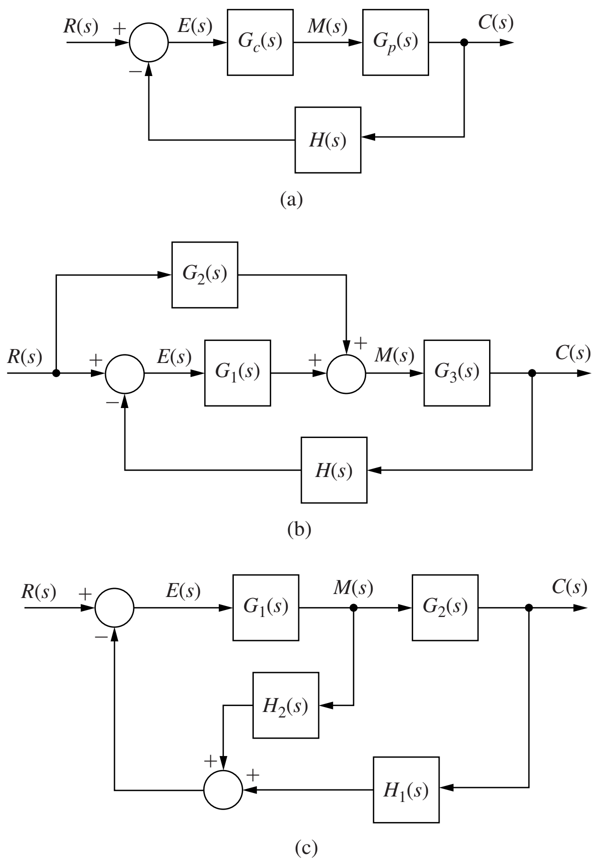


FIGURE P1.1-2 Systems for Problem 1.1-2.

1.1-3. Use Mason's gain formula of Appendix II to verify the results of Problem 1.1-2 for the system of:

(a) Figure P1.1-2(a).
(b) Figure P1.1-2(b).
(c) Figure P1.1-2(c).

1.1-4. A feedback control system is illustrated in Fig. P1.1-4. The plant transfer function is given by

$$G_p(s) = \frac{4}{0.3s + 1}$$

(a) Write the differential equation of the plant. This equation relates $c(t)$ and $m(t)$.
(b) Modify the equation of part (a) to yield the system differential equation; this equation relates $c(t)$ and $r(t)$. The compensator and sensor transfer functions are given by

$$G_c(s) = 20, \qquad H(s) = 1$$

(c) Derive the system transfer function from the results of part (b).

(d) It is shown in Problem 1.1-2(a) that the closed-loop transfer function of the system of Fig. P1.1-4 is given by

$$\frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_P(s)H(s)}$$

Use this relationship to verify the results of part (c).

(e) Recall that the transfer-function pole term $(s + a)$ yields a time constant $\tau = 1/a$, where a is real. Find the time constants for both the open-loop and closed-loop systems.

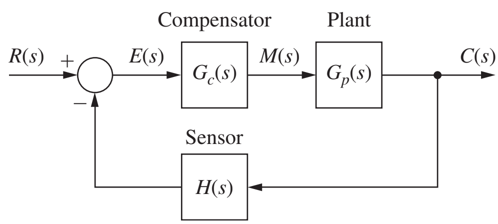


FIGURE P1.1-4 Feedback control system.

1.1-5. Repeat Problem 1.1-4 with the transfer functions

$$G_c(s) = 4, \qquad G_p(s) = \frac{3s + 5}{s^2 + 4s + 4}, \qquad H(s) = 1$$

For part (e), recall that the transfer-function underdamped pole term $[(s + a)^2 + b^2]$ yields a time constant, $\tau = 1/a$.

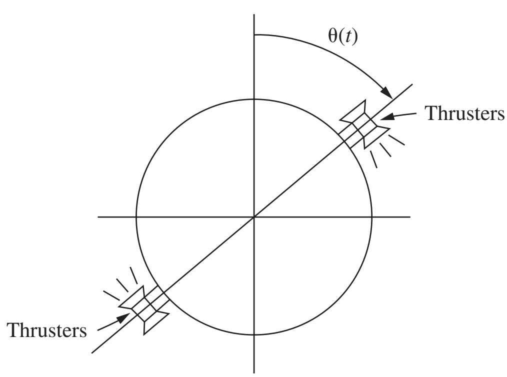
1.1-6. Repeat Problem 1.1-4 with the transfer functions

$$G_c(s) = 4, \qquad G_p(s) = \frac{4}{s^2 + 2s + 1}$$

1.4-1. The satellite of Section 1.4 is connected in the closed-loop control system shown in Fig. P1.4-1. The torque is directly proportional to the error signal.

(a) Derive the transfer function $\Theta(s)/\Theta_c(s)$, where $\theta(t) = \mathcal{L}^{-1}[\Theta(s)]$ is the commanded attitude angle.

(b) The state equations for the satellite are derived in Section 1.4. Modify these equations to model the closed-loop system of Fig. P1.4-1.



(a)

FIGURE P1.4-1 Satellite control system.

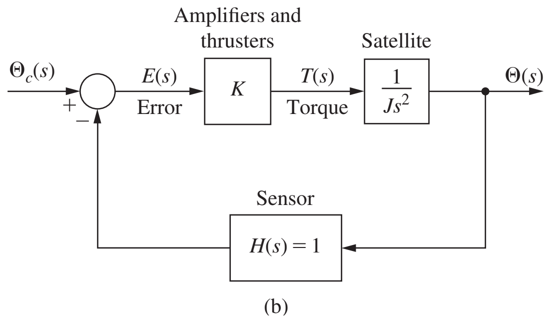


FIGURE P1.4-1 (*continued*)

1.4-2. (a) In the system of Problem 1.4-1, $J = 0.6$ and $K = 12.4$, in appropriate units. The attitude of the satellite is initially at 0°. At $t = 0$, the attitude is commanded to 40°; that is, a 40° step is applied at $t = 0$. Find the response $\theta(t)$.

(b) Repeat part (a), with the initial conditions $\theta(0) = 10°$ and $\dot{\theta}(0) = 30°/\text{s}$. Note that we have assumed that the units of time for the system is seconds.

(c) Verify the solution in part (b) by first checking the initial conditions and then substituting the solution into the system differential equation.

1.4-3. The input to the satellite system of Fig. P1.4-1 is a step function $\theta_c(t) = 4u(t)$ in degrees. As a result, the satellite angle $\theta(t)$ varies sinusoidally at a frequency of 20 cycles per minute. Find the amplifier gain K and the moment of inertia J for the system, assuming that the units of time in the system differential equation are seconds.

1.4-4. The satellite control system of Fig. P1.4-1 is not usable, since the response to any excitation includes an undamped sinusoid. The usual compensation for this system involves measuring the angular velocity $d\theta(t)/dt$. The feedback signal is then a linear sum of the position signal $\theta(t)$ and the velocity signal $d\theta(t)/dt$. This system is depicted in Fig. P1.4-4, and is said to have *rate feedback.*

(a) Derive the transfer function $\Theta(s)/\Theta_c(s)$ for this system.

(b) The state equations for the satellite are derived in Section 1.4. Modify these equations to model the closed-loop system of Fig. P1.4-4.

(c) The state equations in part (b) can be expressed as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\theta_c(t)$$

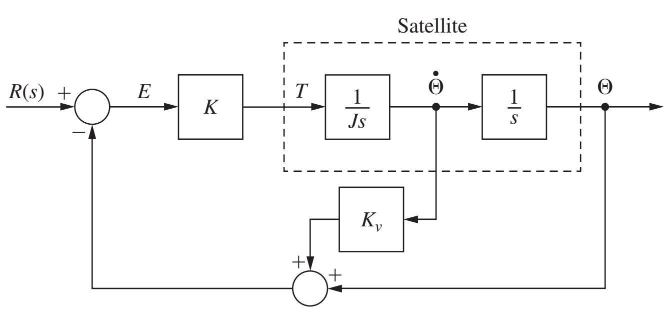


FIGURE P1.4-4 Satellite control system with rate feedback.